Step 3: Effective Birationality

~~私たちはこの general type の定理を経由しないで BAB を直接から攻めて証明するのが目的の1つである~~

次のキーとなる Lemma (by HMX) から始める;

← 基本的には Purtabation テクニック.

<u>Lem 4.</u> (HMX) (X, Δ): log pair of dim n

D: big $\mathbb{R}$-div s.t. $\mathrm{vol}(D) > (2n)^n$

$\Rightarrow$ $\exists V \to B$: a family of subvar. of X s.t.

$x, y \in X$ general, $\exists b \in B$. $0 \leq \exists \Delta_b \sim_{\mathbb{R}} D$

s.t. · $(X, \Delta + \Delta_b)$ is NOT klt at y

· $\exists !$ Non klt place of $(X, \Delta + \Delta_b)$

with the centre $= V_b \ni x$

さらに $B = \cup B_i$ irr. decomp.

$V_i \to B_i$ pullback over B_i

$\to$ $V_i \xrightarrow{dom} X$.

↑ HMX はこれを示したが、実は、Birkar によって、さらにこの V_b が正規でなくても bounded family をなすことまで示した.

HMX の Induction は 上の Lemma 4 の Non klt centre に adjunction を考えれば log general type のカテゴリーで Induction を回す必要があるが、さらに Birkar の Observation により V_b が 正規でなければ Bdd なので、ここが 超 Potentially bound ~~なcentreがbddかどうか~~ の もとの問題に帰することとなる.

↑ ~~ここが1つの BAB と LCT の ACC~~ の違いというか 関係の1つである.

Proof of Lemma 4

$\mathrm{Vol}(D) > (2n)^n$

より

$x \in X$ general pt. に対し

$\exists \Delta_x \geq 0 \sim_{\mathbb{R}} \frac{1}{2}D$ s.t. (X, Δ_x) is NOT lc.

(これは一般へ D の vol > n^n ~> x で multiplicity が高い divisor がとれることから従う)

ヒルベルトスキームを用いて standard な議論

$\longrightarrow$ $\exists \mathcal{D} \to T$ bdd famib of divisors

s.t. : $T \xrightarrow{g} X$ dom

$\forall t \in T$,

$\mathcal{D}_t =: \Delta_t$

(X, Δ_t) is NOT lc

& $\Delta_t \sim_{\mathbb{R}} \frac{1}{2}D$

$x \in X$ genl pt fixして

t のように Δ_t をとっておく.

$y \in X$ general pt.

Pick a $\Delta_s \geq 0$ s.t. $\mathcal{D}_s = \Delta_s$, $s \in T$

& $g(s) = y$.

$\beta := \beta_{s,t} = \sup\{\lambda \in \mathbb{R} \mid (X, \Delta + \lambda(\Delta_t + \Delta_s)) \text{ is lc at } x\}$

s と t とは x と y が同じ genl だから今

$(X, \Delta + \beta(\Delta_t + \Delta_s))$ は x でも y でも NOT klt としてよい

また Perturb をすることで

$(X, \Delta + \beta(\Delta_t + \Delta_s))$ は unique Non klt place

s.t. the center $V_{s,t} \ni x$

としてよい

ここで注意として y は genl なので $\mathrm{Supp}\, D_s \not\ni y$ である.

よってこの perturb で y で Non klt はかわらない.

$B := T \times T$ として

B を高々有限個の l.c. cl. sub set で分解し、それぞれの component で lct が constant になるようにとっておく.

$B = \cup B_i$ として

$$\begin{array}{ccc} V_i & \longrightarrow & B_i \\ \cup & & \in \\ V_{s,t} & \longrightarrow & (s,t) \end{array}$$

となるものが $V_i \to X$ より dom で [illegible] をのぞいて Lem 4 の結論が[illegible]

次の … Prop を上の Lem で用いている

<u>Prop</u> 5 (Birkar, Weichung Chen)

(X, B): d-dim lc pair. D & A nef & big $\mathbb{R}$-divisor.
Assume $D^d > (2d)^d$.

$\Longrightarrow$ $\exists\, \mathcal{P}$: bdd family of sub var of X s.t.

$x, y \in X$ gl closed pts, $\exists\, G_{x,y} \in \mathcal{P}$ $\quad \exists\, \Delta_{x,y} \geq 0 \sim_{\mathbb{R}} D + (d-1)A$

s.t. (1) $(X, B+\Delta)$ is lc at x with unique lc place with centered $G_{x,y}$

(2) $(X, B+\Delta)$ is NOT klt at y

(3) $\dim G = 0$ or $A^{d-\dim G} \cdot G \leq d^d$.

~~bdd family of subvar. of X~~

d

∵ By Lemma 4, $\exists P_0$ s.t. $x, y \in X$ general pt.

$\exists G_0 \in P_0$.

$\exists \underset{\substack{\vee \\ 0}}{\Delta_0} \underset{\mathbb{R}}{\sim} D$

s.t. , $(X, B+\Delta_0)$ is lc at x with unique non klt place centered G_0

. $(X, B+\Delta_0)$ is NOT klt at y.

We prove the following statement:

「 $\forall\, 0 \leq i \leq d-1$

$\exists P_i$ bdd fam of subvar. of X, . $x, y \in X$: general, $\exists G_i \in P_i$

$\exists \underset{\substack{\vee \\ 0}}{\Delta_i} \underset{\mathbb{R}}{\sim} D + iA$

s.t. (1) $(X, D+\Delta_i)$ is lc at x with unique klt place centered G_i

(2) $(X, B+\Delta_i)$ is not klt at y

(3) $\dim G_i \leq d-i-1$ or $A^{d-\dim G_i} \cdot G_i \leq d^d$ 」

iについての帰納法で示す.

i=0のときは $\dim G_0 \leq d-1$ より Done.

$i-1$ までで上の主張が正しいとする

上の G_{i-1} に対して

$\dim G_{i-1} = 0$ or $A^{d-\dim G_{i-1}} \cdot G_{i-1} \leq d^d$

$\longrightarrow$ $G_i := G_{i-1}$ とすればいいので.

$\dim G_{i-1} > 0$ から $A^{d-\dim G_{i-1}} \cdot G_{i-1} > d^{d_i}$ とする.

A: not big ならば $A \sim \underset{\text{ample}}{\underset{\sim}{A_i}} + \underset{0}{\underset{\vee}{E}}$ s.t. $A_i^{d-\dim G_{i-1}} \cdot G_{i-1} > d^d$ とれない

$\leadsto$ (perturbation) $\exists \underset{0}{\underset{\vee}{H}} \sim A_i$, $0 < \delta \ll 1$, $0 < c \ll 1$ s.t.

(1) $(X, B + (1-\delta)\Delta_{i-1} + cH)$ is lc at x with a unique lc place

(2) $(X, B + (1-\delta)\Delta_{i-1} + cH)$ is NOT klt at y centered G'_{i-1}

(3) $\dim G'_{i-1} < \dim G_{i-1}$ $\longleftarrow$ ここから G'_{i-1} は bounded だと分かる. (Lem 4)

$\leadsto$ $G_i := G'_{i-1}$ $\Delta_i := (1-\delta)\Delta_{i-1} + cH + \delta(D + iA) + (1-c)A_i + E_i$

$\sim_{\mathbb{R}} \Delta + iA$.

Done □

ここでのずれの
つうじによって
G'_{i-1} をとって
えらんで
きているが
それこそが
ルートスチーク
を用いて.

早稲田 14回目

Step 3

(1.2) ⟸ (1.1)

Eff ⟹ Pull Compat

Sometime I use $X \xrightarrow{\varphi} Y$, $\mathcal{L}$: $\mathbb{Q}$-div.
When $\varphi^* \mathcal{L}$ is well-def, we denote $\mathcal{L}|_X$

Prop 4.5 = Thm 1 ← 実際は lower dim の ACC : global ε' + δ'.

$d \in \mathbb{N}$, $\varepsilon, \delta \in \mathbb{R}$

$\Rightarrow \exists m \in \mathbb{N}$ s.t. $\forall X$: ε-lc Fano var of dim d s.t. · $K_X + B \sim_{\mathbb{R}} 0$ · $B \geq \delta$

$\Rightarrow |-mK_X|$ defines birational map

Outline of proof By contradiction, take (X_i, B_i) as above

m_i : the minimal # of $|-m_i K_{X_i}|$ defining birational map

satisfies $m_i \nearrow \infty$

On the other hand, we make have non klt pair $(X_i, \frac{1}{\delta} B_i)$

Always we try to contradiction to the following Prop.

(Some version of open condition of klt pair)

Prop 4.2 $d \in \mathbb{N}$, $\varepsilon > 0$ $\mathcal{P} := \{(W_\lambda, D_\lambda) \mid$ W_λ : d-dim sm. proj. var, D_λ : SNC reduced div, $\lambda \in S$ finite type scheme $\}$

= Prop 2 i.e. log bounded family

$\Rightarrow \exists \delta > 0$ s.t.
- (W, T) : ε-sub lc
- T: reduced S.N.C divisor on W & $(W, \operatorname{Supp} T) \in \mathcal{P}$
- $B_W \geq 0$ $\mathbb{R}$-Cartier on X
- $\operatorname{Supp} B_W$ S.N.C
- $\exists N$ with $B_W \sim_{\mathbb{R}} N$ & $\operatorname{Supp} N \subseteq T$, $-\delta \leq N \leq \delta$

$\Rightarrow$ $(W, T + B_W)$ is sub KLT.

②

Note that Original Prop 4.2 is more genl but in this talk

I use this formulation

Check in curve case $\deg N = \deg B_W \leq \delta \cdot$ (# of comp. of T)

Take $0 < \delta \ll 1 \rightsquigarrow$ o.k.

言わない方がよい.
(general case: Now this is SNC. coefficients become
So just cutting by hyperplanes we can reduce to curve case.)

So main problems are always

① Can we construct bdd bounded family?
② Complement boundary B_W can be $\sim_{\mathbb{R}}$ sum of very small coeff of comp in the bounded family?

We

discuss 2-fold contradictions arguments

let n_i s.t. $\mathrm{Vol}(-n_i K_{X_i}) > (2d)^d$ (Later change it to minimal one.)

Step 1 $\quad \dfrac{m_i}{n_i} <$ bdd

in this step, we argue ① ②

& make contradiction to prop ~~4~~. 2. (Prop 2)

Under assuming Step 1, we ~~ag~~ argue again ① ② & produce contradiction
Choose n_i as the minimal one. i.e. $\mathrm{Vol}(-(n_i-1)K_{X_i}) \leq (2d)^d$ to prop 4.2.

$\rightsquigarrow \mathrm{Vol}(-m_i K_{X_i}) <$ bdd

(ここで使うは $m_i/n_i <$ bdd より n_i/m_i を加える.)

$\because \mathrm{Vol}(-m_i K_{X_i}) = \left(\dfrac{m_i}{n_i-1}\right)^d \mathrm{Vol}(-(n_i-1)K_{X_i})$

$\leq (2r)^d \cdot (2d)^d$

No. 14-② (ここにつかう)

Date

<u>Claim</u> $\frac{m_i}{l_i} < {}^{\exists}n$.

$\because$ 今、$\frac{l_i}{n_i} < {}^{\exists}a$ と仮定すると

n_i の代わりに $d a n_i$ について Lem 4 をつかうと.(ここは Prop 5 の証明と同じ様に行う)

G_i の次元をおとすことができる.
よっていつか $\dim G_i = 0$ となり, $d a n_i K_{X_i}$ は potentially broad
となり上のギロンと同様に矛盾する.

よって $\frac{l_i}{n_i} \nearrow \infty$ としてよい.

いま $\frac{m_i}{l_i} \nearrow \infty$ とする $\longrightarrow$ $\frac{m_i}{n_i + l_i} = \frac{\frac{m_i}{l_i} \nearrow \infty}{\underset{\downarrow 0}{\frac{n_i}{l_i}} + 1} \nearrow +\infty$

よって n_i を $n_i + l_i$ と変えることで.

$$\frac{l_i}{n_i + l_i} = \frac{1}{\frac{n_i}{l_i} + 1} < \text{bdd}$$

の case に帰着されてしまう

よって $\frac{m_i}{l_i} \not\nearrow \infty$ と仮定できる. これが Claim である.

Note that Original Prop 4.2 is more genl but in this talk

I use this formulation

∴ Check in curve case $\deg N = \deg B_W \le \delta \cdot (\#\text{ of comp. of } T)$

Take $0 < \delta \ll 1 \rightsquigarrow$ o.k.

言わない方がいい.

(general case Now this is snc coefficients become

So just cutting by hyperplanes we can reduce to curve case)

So main problems are always

① Can we construct bdd bounded family?

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We

discuss 2-fold arguments

contradictions

let n_i s.t. $\mathrm{Vol}(-n_i K_{X_i}) > (2d)^d$ (Later change it to minimal one.)

Step 1

$$\frac{m_i}{n_i} < \text{bdd}$$

in this step, we argue ① ②

& make contradiction to prop 4.2. (Prop 2)

Under assuming Step 1, we ~~as~~ argue again ① ② & produce contradiction to prop 4.2

Choose n_i as the minimal one. i.e. $\mathrm{Vol}(-(n_i-1)K_{X_i}) \le (2d)^d$

$\rightsquigarrow \mathrm{Vol}(-m_i K_{X_i}) < \text{bdd}$

(ここでじつは $m_i \nearrow \infty$ or $n_i \nearrow \infty$ を使う.)

$$\because \mathrm{Vol}(-m_i K_{X_i}) = \left(\frac{m_i}{n_i - 1}\right)^d \mathrm{Vol}(-(n_i-1)K_{X_i})$$

$$\le (2r)^d \cdot (2d)^d$$

(Step 2 はこれで証明終である)

Step 2 bdd of vol$(-m_iK_{X_i})$ $\Longrightarrow$ by bddness of the follows.

$W_i \xrightarrow{\phi_i} X_i$ ... log res of (X_i, B_i)

s.t. $\phi_i^*(-m_iK_{X_i}) \sim_{\mathbb{Z}} \underbrace{A_{W_i}}_{\text{bpf}} + \underbrace{R_{W_i}}_{\text{Fixed part}}$

$\Sigma_{W_i} := \underbrace{E_{W_i}}_{} + \underbrace{A_{W_i}}_{\text{genl member}}$

E_{W_i}: total except div. of ϕ_i

$$W_i \xrightarrow{\phi_{|A_i|} = \psi_i} \tilde{W}_i \qquad \Sigma_{\tilde{W}_i} := \psi_{i*}\Sigma_{W_i}$$

$W_i \to X_i$

ただし、$W_i \to X_i$ は[illegible] 正規化に

Goal $(\tilde{W}_i, \Sigma_{\tilde{W}_i})$ is log bounded. $\rightarrow$ we may assume $W_i \to \tilde{W}_i \to$ [illegible] ここが log bdd

同次特殊 [illegible]

$\{(W_i, \Sigma_{W_i})\}$ is also bounded

(in Thm (HMX) in the Jiang's SC.) ← ここで ACC (by ...) つかってないよー

ETS $\mathrm{vol}(W_i, \underbrace{K_{W_i} + \Sigma_{W_i} + 2(2d+1)A_{W_i}}_{\text{true big by length of extremal ray}}) < \underline{\text{bdd}}$

(東大の講演の 2-③ step1 と同じである)

But this is easy: $\parallel$

$$\mathrm{vol}(K_{\tilde{X}_i} + \Sigma_{\tilde{W}_i} + 2(2d+1)A_i) \leq \mathrm{vol}(K_{X_i} + R_i + (4d+3)A_i)$$

$$= \mathrm{vol}\,\phi_{i*}(\quad) \leq \mathrm{vol}((4d+3)R_i + (4d+3)A_i)$$

$$\leq \mathrm{vol}((4d+3)(-m_iK_{X_i})) \leq \text{bdd}$$

Step 3 The coefficients $A_W + R_{W_i}$ is bounded from the above

(the same arguments is included in the proof of Step 1)

一回みとめる

$\rightsquigarrow 0 < \frac{1}{m_i\delta} A_{w_i} + \frac{1}{m_i\delta} R_{w_i} << 1$

$S\,\mathbb{R}$

$\frac{1}{\delta} B_i|_{W_i}$

$\rightsquigarrow$ $(W_i, \frac{1}{\delta}B_i|_{W_i})$ is klt by Prop 4.2 (Prop 2) ... ↯

Thus we show step 1 $\frac{m_i}{n_i} <$ bdd. (Prop 4.4)

Note that we do NOT assume the minimality of n_i.

So sometimes we change n_i to n_i' ($n_i' > n_i$) under satisfying some condition.

by contradiction we may assume $\frac{m_i}{n_i} \nearrow \infty$ (in particular $m_i \nearrow \infty$)

$\phi_i : W_i \to X_i$ same as in Step 2

$\downarrow$

$\hat{W}_i$ $R_i := \phi_{i*} R_{W_i}$

Strategy of the proof: $A_i := \phi_{i*} A_{W_i}$

Step A (Take subadjunction) exceptional $\exists!$ lc place.

Find $G_i \subseteq X$ s.t. • G_i: lc centre of (X_i, Δ_i)

• $\mathrm{vol}(-m_i K_{X_i}|_{G_i}) <$ ${}^{-n_i K_{X_i}}$ bdd.

Step B

$F_i \xrightarrow{\nu_i} G_i$ the normalization $\qquad (K_{X_i}+\Delta_i)|_{F_i'} = K_{F_i}+\Delta_{F_i}$

$\uparrow \qquad \ni x_i$

$F_i' \subseteq W_i$

$\rightsquigarrow (F_i', \mathrm{Supp}\,\Delta_{F_i})$ is $\overset{\text{bir}}{\text{bdd}}$ (essentially same arguments as Step 2)

Step C Bounds coefficient of Δ_{F_i} ≒ Step 3.

$\rightsquigarrow$ by Adding $\underset{\sim}{\frac{1}{3}B_i}|_{F_i}$, $\longrightarrow$ ε to Prop 4.2 (Prop 2)

↑ by the genericity of x & y, this is well defined.

Step A

Prop (, Kollár)

(1) D: ample s.t. $\mathrm{vol}(D) > (2d)^d$

$\Rightarrow$ $x, y \in X$ gen $\exists\, \Delta \sim_{\mathbb{Q}} D$ s.t. $\Delta \geq 0$

up to x & y possibly change,

· (X, Δ) is LC ~~KLT~~ at x & NOT LC at y

· $\exists$ exceptional LC center $G \ni x$

(cf. HMX Lemma 2.3.3)

(2) $\dim G = 0 \Rightarrow |K_X + \lceil (1+\varepsilon) D \rceil|$ defines bir. map for $\forall \varepsilon > 0$

(3) when $\dim G > 0$

if A: ample $\mathbb{Q}$-div s.t. $\mathrm{vol}(A|_G) > d^d$ up to x & y changing

$\Rightarrow$ $x, y \in X$ gen $\exists\, \Delta' \sim_{\mathbb{Q}} \Delta + A$ s.t. · $\exists\, G'$ min in the pair of G

· $G' \subsetneq G$

We apply it for $D = -n_i K_{X_i}$

<u>Now</u>

[$\mathrm{vol}(-n_i K_{X_i}) > (2d)^d$, すり] わからないの？

~~~~ or: $\dim G_i = 0 \Rightarrow |K_{X_i} - (2n_i+1) K_{X_i}|$ defines birat.

$$= \frac{2n_i+1}{n_i} n_i$$

$$\frac{2n_i+1}{n_i} = 1 + (1 + \frac{1}{n_i})$$

$\longrightarrow 2n_i \geq m_i$ ↯

We may assume $\dim G_i > 0$

<u>Note</u> : G_i's form covering family.

So this means we may assume that the general fibre of this family has positive dimension.

So we may assume $\dim G_i = 0$ or $\mathrm{vol}(-n_i K_{X_i}|_{G_i}) < d^d$

In the latter case

for the family of G_i's. by Prop (Prev)

$l_i :=$ the smallest # st $\mathrm{Vol}(-l_i K_{X_i}|_{G_i}) > d^d$.

Claim we may assume $\frac{m_i}{l_i} \leq \mathrm{bdd}$

∵ $\frac{l_i}{n_i}$: unbounded

をまず示す

$\frac{l_i}{n_i} < \mathrm{bdd} \to \frac{m_i}{n_i + l_i} \nearrow \infty$ より $n_i \rightsquigarrow n_i + l_i$ とる.] ここは ~~本~~ 18年度の講究ノート 14-③ ページ をみた方がいい

We may assume $\frac{l_i}{n_i} \nearrow \infty$

If we assume $\frac{m_i}{l_i} \nearrow \infty$ then $\frac{m_i}{n_i + l_i} = \frac{\frac{m_i}{l_i}}{\frac{n_i}{l_i} + 1} \nearrow \infty$

より change $n_i \rightsquigarrow n_i + l_i$ とる
~~~~

(7)

したがって $\mathrm{vol}(-m_i K_{X_i}|_{G_i}) = \left(\frac{m_i}{l_i-1}\right)^{d'} \mathrm{vol}(-(l_i-1)K_{X_i}|_{G_i})$

$$\leq \left(\frac{m_i}{l_i-1}\right)^{d} d^{d} \in \mathrm{bdd}$$

$d' = \dim G_i$

Step A ok

Step B $\mathrm{Vol}(-m_i K_{X_i}|_{G_i}) < \mathrm{bdd} \Rightarrow \delta_2$ bir. bdd

More precisely we need to subadjunction statement

Thm (Kawamata's subadjunction $+\alpha$)

$G \subseteq X$ & (X, B) klt $\Delta \geq 0$ $\Delta \in$ DCC set I

If G is Exceptional lc center of $(X, \Delta + B)$

then (I) $F \to G$ normalization

$$(K_X + \Delta)|_F = K_F + \Theta_F + P_F$$

Θ_F: DCC set, P_F: p.e.

この分解の Θ は ② の下3行 (Supp) ではー方ーは 1 で与える。

(II) $M \geq 1$: $\mathbb{Q}$-Cartier $\mathbb{Q}$-div on X

$G \nsubseteq \mathrm{Supp}\, M$

$\rightsquigarrow \mu_P(\Theta_F + M|_F) \geq 1$ for $\forall P \subseteq M|_F$

(III) G is a member of a cov. family of X

then $K_X|_F = K_F + {}^{\exists}\Delta_F$

s.t. (F, Δ_F) : sub ε-lc if X: ε-lc

$\Delta_F \leq \Theta_F$

thanks to 2004 JB

$$\begin{array}{ccc} F_i' & \xrightarrow{\text{sm}\ \to P_i\ \to} & G_i \\ & & \cap \\ W_i^0 & \longrightarrow & X_i \end{array}$$

subadj そこをPiと ⊕ごとか P_i とか大 & $M_i := \underset{\beta F_i F_i}{\underset{\sim}{A_{w_i}}} + \underset{P_{i\,\text{wo}}}{\overline{R_{w_i}}}$

$\Sigma_{F_i'} := \cup$ comp of $\begin{cases} \cdot\ \text{bir. trnsfrm of } \Theta_i|_{F_i} \\ \circ\ M_i|_{F_i'} \\ \circ\ \text{excep}/F_i' \end{cases}$

As same as the step2

ETS $\mathrm{vol}(K_{F_i'} + \Sigma_{F_i'} + 2(2d+1)A_{w_i}|_{F_i}) \leq \mathrm{bdd}$

$\to (F_i', \Sigma_{F_i'})$: lg bound bdd.

To show this, we introduce the follwg div:

$\cdot H_{F_i'} \in_{\text{gen}} |6d' A_{w_i}|_{F_i}|$, $0 < \varepsilon' < \min_{\substack{\uparrow \\ \text{DCC}}} \frac{1}{2}, \Theta_{F_i} \cup \{\frac{\varepsilon}{2}\}$

$\Omega_{F'} :=$

$\mu_D \Omega_{F_i'} := \begin{cases} \varepsilon' & D: \text{comp of bir trs of } \Theta_i|_{F_i}\ \&\ M_i|_{F_i'} \\ \frac{1}{2} & \text{others} \end{cases}$

& require ~~F_i~~ $\mathrm{Supp}\,\Omega_{F_i'} = \mathrm{Supp}(\Sigma_{F_i'} + H_{F'})$

$\longrightarrow (F_i', \Omega_{F_i'})$: ε-lc $\left[\ \&\ K_{F_i'} + \Omega_{F_i'} \text{ big by cone thm} \right]$

$-\varepsilon' \geq -1+\varepsilon'$

$-\frac{1}{2} \geq -1+\varepsilon'$

言わない(いい)

Claim $\mathrm{vol}(K_{F_i'} + \Omega_{F_i'}) \leq \mathrm{bdd}$

$\wedge|$

$\mathrm{vol}(K_{F_i} + \Omega_{F_i})$ ← F_i... push fwd thanks to the def of $\Omega_{F_i'}$

$\wedge|$ ⊗

$\mathrm{vol}(K_{F_i} + \Delta_{F_i} + 5d' \underset{A_i + R_i}{M_i}|_{F_i})$ $\boxed{\left(\frac{n_i - 1 + d'}{m_i}\right)^{d'}}$

$\|$

$\mathrm{vol}(-(n_i - 1 + 5d' m_i)K_{X_i}|_{F_i}) = \mathrm{vol}(-m_i K_{X_i}|_{F_i}) < \mathrm{bdd}$

今、$\mathrm{vol}(-(K_{F_i'}+\Omega_{F_i'})) < bdd \Rightarrow \mathrm{vol}(K_{F_i'}+\Sigma_{F_i'}+2(d'+1)A_{W_i}|_{F_i'})$

$< bdd$ を見る

(∵) ACC for pseudo-effective threshold ← $-d-1$次元以下のACCでOK.

$\exists \alpha < 1$ s.t. $K_{F_i'} + \underset{\text{independent of } i}{\alpha}\, \Omega_{F_i'}$: big

$$\mathrm{vol}(K_{F_i'}+\Sigma_{F_i'}+\underbrace{2(d'+1)A_{W_i}|_{F_i'}}_{\substack{\wedge|\\ 4(2d'+1)\Omega_{F_i'}}}) \le \mathrm{vol}(K_{F_i'}+\Omega_{F_i'}+p(1-\alpha)\Omega_{F_i'})$$

D : $\Sigma_{F_i'}$ の comp.

$\mu_D(\Omega_{F_i'}) \ge \varepsilon'$

ならば $p = \dfrac{3(2d'+1)}{(1-\alpha)\varepsilon'}$ くらいとっておけば...

$$\le \mathrm{vol}(K_{F_i'}+\Omega_{F_i'}+p(K_{F_i'}+\alpha\Omega_{F_i'})+p(1-\alpha)\Omega_{F_i'})$$

$$\le \mathrm{vol}((1+p)(K_{F_i'}+\Omega_{F_i'})) \le bdd \quad \square \longrightarrow \text{したがって } (X_i, \Sigma_i) \text{ は log bdd}$$

Step C (= Step 3)

Claim $M_{F_i'} \le bdd$

$\| $

$A_{F_i'} + R_{F_i'} = A_{W_i}|_{F_i'} + R_{W_i}|_{F_i'}$ ← doesn't depends on i (by bdd family etc)

(∵) Take $B_{F_i'}$: ample Cartier & $b \ge 0$

s.t. $bM_{F_i'} - B_{F_i'}$ big

$$\underbrace{M_i|_{F_i'}}_{\text{nef}} \cdot \underbrace{(B_{F_i'})}_{\text{ample}}^{d'-1} \le \mathrm{vol}(M_i|_{F_i'}+B_{F_i'}) = \mathrm{vol}((1+b)M_i|_{F_i'})$$

$= \mathrm{vol}(-(1+b)\phi_i^* K_{X_i}|_{F_i'})$

$= \mathrm{vol}(-(1+b)K_{X_i}|_{G_i}) < bdd$ ☐

時間があまったら

☆の不等式の証明

ETS $\Theta_i + 5d \overset{\text{push of } M_i}{M_{F_i}} - \overset{\text{\& } \Omega_{F_i}'}{\Omega_{F_i}}$ big (P_i は p.e. だから)

Note $\mu_D(\Theta_i + M_i) \geq 1$ for $D \subseteq \underbrace{M_i}_{\text{integral div}}$ by (II) in subadjunction

D のケースを比較する

Case $D = \underset{\text{push of } H_{F_i}'}{H_{F_i}}$ or $D \subseteq M_{F_i}$ comp

$\longrightarrow \mu_D \Omega_{F_i} = \frac{1}{2} < 1 \leq \mu_D(\Theta_i + M_i)$

Case D: comp of Θ_i & $D \subseteq \operatorname{Supp} M_{F_i}$

$\mu_D \Omega_{F_i} \leq \mu_D \Theta_{F_i} \leq \mu_D(\Theta_{F_i} + M_i)$

$\Omega_{F_i} - \frac{1}{2} H_{F_i} \leq \Theta_i + M_{F_i}$

$4dM_{F_i} - \frac{1}{2} H_{F_i} \sim 4dA_i|_{F_i} + 4dR_i|_{F_i} - \underset{3dA|_{F_i}}{\overset{\wedge|}{3dA_{F_i}}}$ big

$\underbrace{\Theta_{F_i} + M_{F_i} - (\Omega_{F_i} - \frac{1}{2} H_{F_i})}_{\geq 0} + \underbrace{4dM_{F_i} - \frac{1}{2} H_{F_i}}_{\text{big}}$ big

$\|$

$\Theta_{F_i} + 5dM_i - \Omega_{F_i}$ ✓