Pluricanonical representations and abundance conjecture

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⋄ My research interest

I am interested in birational geometry and Hodge theory. Especially in the minimal model theory and canonical bundle formulae.

⋄ Research motivation

In studies on higher dimensional algebraic varieties, the following conjecture is very important:

Abundance conjecture

X: proj. mfd.

if K_X is nef then K_X is semi-ample.

We sometimes need to generalize it for more bad singularities or log pairs (e.g. klt, lc, etc...) for approach to it for even smooth varieties.

⋄ Our results

This is a joint work with O. Fujino at Kyoto Univerisity.

We work over the complex number field $\mathbb{C}$.

Main Theorem [Fujino-G, '11]

(X, Δ): proj. slc pair,

$\nu : (X', \Theta) \to (X, \Delta)$:the noramalization,

where $\nu^*(K_X + \Delta) = K_{X'} + \Theta$.

Then $K_X + \Delta$ is semi-ample if so is $K_{X'} + \Theta$.

Definition[Semi-log canonical]

X: red., S_2, pure dim., and n.c. in codim. 1.

$\Delta \geq 0$: $\mathbb{Q}$-div. s.t. $K_X + \Delta$ is $\mathbb{Q}$-Car.

$X := \bigcup X_i$: irr. decom.,

$\nu : X' := \coprod X'_i \to X = \bigcup X_i$: normalization.

Define Θ by $K_{X'} + \Theta = \nu^*(K_X + \Delta)$.

$\Theta_i := \Theta|_{X'_i}$.

(X, Δ) is *semi-log canonical (slc)* $\Leftrightarrow$ (X'_i, Θ_i): lc for every i.

We believe that the above main theorem is one of the important steps of approaching the abundance conjecture.

The following corollary also seems to be important:

Corollary [cf. Keel–Matsuki–McKernan, '94]

$\exists$ good minimal models for klt pairs in dim. n.

$\Rightarrow$ Abundance conj. for lc pairs in dim. n.

⋄ Known Results on Main Thm.

∘ Kawamata, Abramovich–Fong–Kollár–McKernan'92 in dim. 2

∘ Fujino'00 in dim. 3

∘ G'10 when $K_X + \Delta \equiv 0$

⋄ Key Thm. in the proof of Main Thm.

It's sufficient to show the following by Fujino'00's argements and Birkar–Cascini–Hacon–McKernan's MMP:

Theorem

(X, Δ): proj. lc pair with base point free line bundle $m(K_X + \Delta)$.

Then $\rho_m(\mathrm{Bir}(X, \Delta))$ is a finite group

,where

Definition[Log pluricanonical representation]

(X, Δ): pair.

prop. bir. map $f : (X, \Delta) \dashrightarrow (X, \Delta)$ is *B-birational* $\Leftrightarrow$ $\exists$ resol. $\alpha, \beta : W \to X$ such that $\alpha^*(K_X + \Delta) = \beta^*(K_X + \Delta)$ and $\alpha = f \circ \beta$.

$\mathrm{Bir}(X, \Delta) := \{\sigma \,|\, \sigma : (X, \Delta) \dashrightarrow (X, \Delta) \text{ is } B\text{-birational}\}$.

We call

$$\rho_m : \mathrm{Bir}(X, \Delta) \to \mathrm{Aut}_{\mathbb{C}}(X, m(K_X + \Delta))$$

a *log pluricanonical representation* of $(X.\Delta)$.

Remarks

∘ For klt pairs, Theorem is true for the bigger group than $\mathrm{Bir}(X, \Delta)$ without the assumption that $K_X + \Delta$ is semi-ample. And we need the generalization for proving Theorem for lc pairs.